Theoretical Analysis - Speed of Response and Proportionality of DVE and the 3rd MFC to the Driving Trace

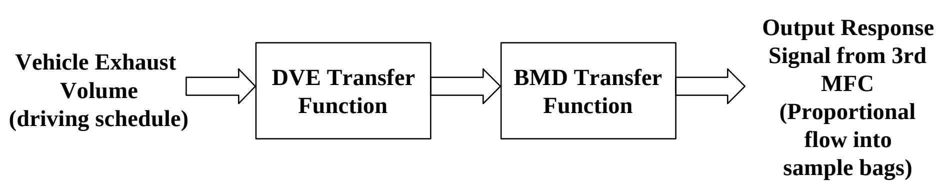
Introduction

A key requirement to the proper functioning of the bag mini-diluter system is the proportional filling the sample bags to the vehicles exhaust volume. As previously mentioned this is accomplished through the use of a 3rd MFC whose flow rate is varied throughout the test schedule and is controlled by the DVE flow rate signal. To demonstrate the bag mini-diluters ability to do this we will next explore the time response of the MFC's and the DVE, and their relationship to the driving trace. A theoretical model will be developed using these time constants and then compared to actual vehicle test data.

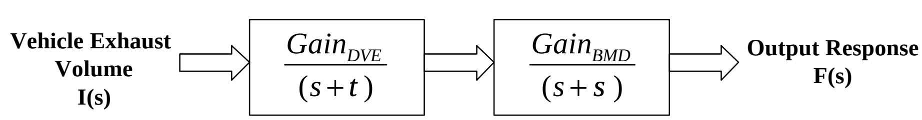
Theoretical Model

A theoretical model of the DVE/BMD system was developed to demonstrate proportionality of the 3rd MFC (inside the BMD) as compared to the exhaust volume signal. Of primary focus were the known time delays in two devices, namely the DVE and the 3rd MFC. There may be other interactions not encompassed in this model.

The BMD uses the DVE to control the 3rd MFC thereby providing proportionality. Using classical control systems theory, the input signal ("Vehicle Exhaust Volume") through to output signal ("Output Response from 3rd MFC"), could be modeled as follows:



Or in control systems (Laplace Transform) format



Model for DVE and BMD System

This model assumes the DVE transfer function can be modeled as a single pole (active) low pass filter, with time constant τ and gain $G_{DVE..}$ The actual control system may be much more complicated inside the DVE itself, but this model is focusing on the over all performance of the device from a time delay perspective.

Like wise it was assumed that the Bag Mini-diluters control of the 3rd (proportional) MFC is also a single pole (active) low pass filter, with time constant σ and gain G_{BMD}.

Speed of Response:

A critical element in determining the overall systems performance of the model is to determine the speed of response for all the individual components used. The two major devices used are the DVE and the 3rd (or proportional) MFC inside the bag mini-diluter itself. The DVE has an advertised response time of 0.5 seconds (0 – 90% of full-scale) (reference EFLOW DVE specification). The bag mini-diluters 3rd (or proportional) MFC has an advertised speed of response of 1.2 seconds, assumed for 0 to 90% FS (reference Porter MFC specification “Fast Response Mass Flowmeter and Controller Series 100F and 200F – Technical and Users Manual”, FM-369 Rev D 2/97). For this analysis it is assumed that the control PC computer’s does not significantly impact the system speed of response.

Because the DVE and the 3rd MFC’s speed of response is reported from 0 to 90% of full scale, we need to convert these times into classical control systems (base e) time constants of 0 to 63.2% of full scale. Looking at the base e equation, multipliers on time constants return the following responses:

$e^{-1} \Rightarrow$ 63.2% of full scale
$e^{-2} \Rightarrow$ 86.4% of full scale
$e^{-3} \Rightarrow$ 95.2% of full scale

$e^{-2.3}$ yields the correct 0-90% of full scale response. Using this information we can estimate the true (exponential) time constants of the DVE and 3rd MFC by dividing the response time by this 2.3 factor as follows:

Device	0-90% Response Time	Estimated Equivalent Time Constant (τ)
DVE	0.5 seconds	0.22 seconds
3rd MFC	1.2 “	0.52 “

Next, to look at whether these speeds are fast enough to follow the vehicles operating modes we also need to determine the actual vehicle exhaust volume speed of response. Since we can’t measure exhaust volume directly (w/o a DVE instrument), the best way to look at exhaust volume fluctuations is to focus on the required driving trace and see if the DVE/Bag mini-diluter system can follow the schedule. This assumes the vehicle exhaust volume will respond no faster than the driving trace “input signal”.

In driving schedules acceleration rate is important here because it is difficult to determine the driving schedules “time constant”. Theoretically the schedule is updated at a 10 hertz rate, but the actual schedule, driver and vehicle respond much slower than 10 hertz. In reality the real time response is probably not much faster than 0.5 to 1 second (2 to 1 hertz). It is more appropriate to characterize the driving schedule as a series of ramps with differing slopes, versus the classical control systems approach of unit steps or impulses. Using this model, we can characterize a simple driving schedule acceleration as a unit ramp function with gain G_{Ramp} as follows:

$$DrivingTrace = Accel_{RATE} * t$$

Where
$$Accel_{RATE} = AccelerationRate(inMPH/\sec)$$

Taking the Laplace transform of this input function yields:

$$DrivingTrace(s) = \frac{Accel_{RATE}}{s^2}$$

To determine a worst case acceleration rate for the various driving schedules, the US06 driving schedule was used. Clearly the US06 driving schedule have some of the more aggressive accelerations used in the industry today. Looking at several of the US06 acceleration modes and calculating the acceleration rate for these cycles, an average ramp or acceleration rate can be calculated of about 3.7 mph/sec. This was based on a sampling of several individual mode data as shown below:

Acceleration Mode	Speed Grade	Elapsed Time	Acceleration
US06 - Cycle 1	0 - 44 mph	15 seconds	2.9 mph/sec
“ - “ 2	0 - 55 “	16 “	3.4 “ “
“ - “ 3	0 - 53 “	17 “	3.2 “ “
“ - “ 4	0 – 28 “	6 “	4.6 “ “
“ - “ 5	0 – 52 “	11 “	4.6 “ “
Approximate Average Acceleration Rate =			3.7 mph/sec

Theoretical Response of DVE/BMD System – Numerical Analysis

Once the model is established and the critical parameters have been determined, the Output Response of the 3rd MFC can be calculated as the product of the DVE, 3rd MFC and Input transfer functions as follows:

$$Output\,Response(s) = I(s) * H(s)$$

where the input ramp function I(s) is:

$$I(s) = \frac{Accel_{Rate}}{s^2}$$

and the BMD/DVE function H(s) is:

$$H(s) = \frac{Gain_{DVE}}{(s+t)} * \frac{Gain_{MFC}}{(s+s)}$$

Combining these two terms yields and Output Response of:

$$Output\,\mathrm{Re}\,sponse(s) = \frac{Accel_{Rate}}{s^2} * \frac{Gain_{DVE}}{(s+t)} * \frac{Gain_{MFC}}{(s+s)}$$

where:

$$Accel_{Rate} = 3.7 mph/\sec$$

$$Gain_{DVE} = t$$

$$Gain_{MFC} = s$$

τ and σ were used as gain terms in this analysis to make the low pass filters active with unity gain. In reality these coefficients would be set to yield engineering units of exhaust volume and the appropriate bag fill rate. For this analysis, these gain terms are not of importance. Using these terms yields:

$$Output\,\mathrm{Re}\,sponse(s) = \frac{3.7}{s^2} * \frac{t}{(s+t)} * \frac{s}{(s+s)}$$

Partial fraction expansion yields:

$$= \frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s+t)} + \frac{D}{(s+s)}$$

where:

$$A = \left[\frac{d}{ds}\left(s^2 * Output\,\mathrm{Re}\,sponse(s)\right)\right]_{s=0}$$

$$= \frac{-3.7 * (t+s)}{t\,s}$$

$$B = \left[s^{2*} * Output\,\mathrm{Re}\,sponse(s)\right]_{s=0}$$

$$= 3.7$$

$$C = \left[(s+t)*Output\,\mathrm{Re}\,sponse(s)\right]_{s=-t}$$
$$= \frac{3.7*s}{t(s-t)}$$

$$D = \left[(s+s)*Output\,\mathrm{Re}\,sponse(s)\right]_{s=-s}$$
$$= \frac{3.7*t}{s\ (t-s)}$$

Taking the inverse Laplace transform of this equation yields:

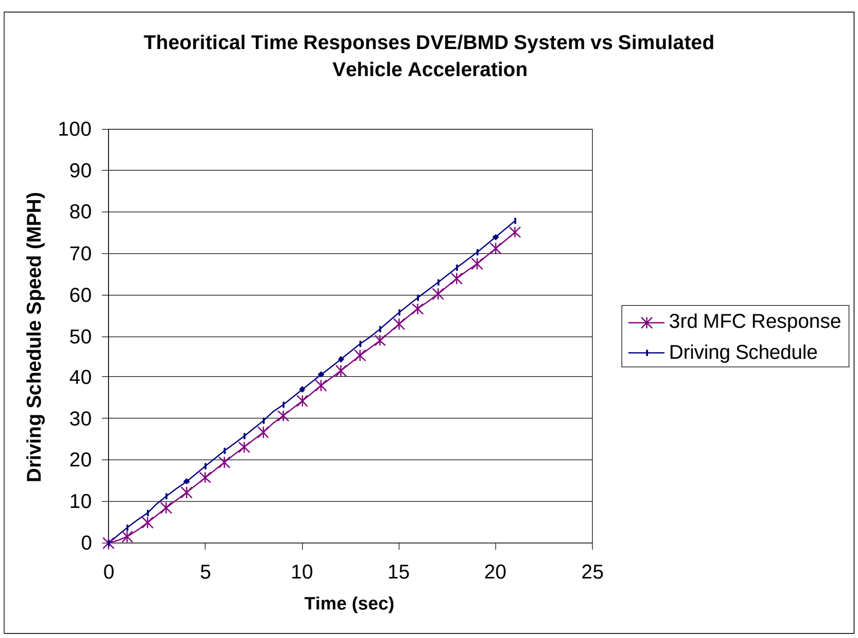
$$L^{-1}(Output\,\mathrm{Re}\,sponse(s)) = L^{-1}\left[\frac{A}{s}\right] + L^{-1}\left[\frac{B}{s^2}\right] + L^{-1}\left[\frac{C}{(s+t)}\right] + L^{-1}\left[\frac{D}{(s+s)}\right]$$

Converting this equation to the time domain yields:

$$Output\,\mathrm{Re}\,sponse(t) = -3.7*\left[\frac{u(t)*(t+s)}{ts} + t + \frac{s*e^{-tt}}{t(s-t)} + \frac{t*e^{-st}}{s(t-s)}\right]$$

A reference for this equation can be found in Automatic Control Systems Engineering; Volume 1; author A.W. Langill, jr; Prentice Hill 1965; Appendix A: Table of Laplace Transforms. U(t) represents a unit step input that starts at time t = 0.

A numerical analysis was performed using this equation, then the Output Response (3rd MFC flow rate signal) was plotted against the input signal (driving trace ramp) as shown below.



Analysis of this theoretical graph shows for this simulated (ramp) acceleration, that the DVE/BMD system does follow the input signal fairly well, but lagging a small amount. The theoretical lag is equal to the summation of the time constants involved. In this example the time lag would be approximately 0.74 seconds (0.22+0.52). However even with the time lag involved, it remains **proportional** to the input signal

One way to interpret this data is that as long as the time constants of the DVE/BMD are on the same order (or faster than) the input signal "frequency" response, then the two will correlate fairly well.

Actual (Measured) DVE/BMD Time Response Data

To show this ability of the 3rd MFC to track the DVE, a series of 4 actual vehicle tests were run, FTP #1, FTP #2, US06 #1 and US06 #2. This data is attached. During these vehicle tests the DVE analog output (measured exhaust flow) and the BMD's 3rd MFC feedback signal (proportional bag fill flow rate) were measured. The DVE unit used was the production unit (not the prototype unit used for the 1996, through 1997 vehicle testing). It was felt that the production unit would be more representative of units used for future testing.

The 1-second data was plotted for the whole test plus zoomed-in plots were made in 250 second increments. To demonstrate proportionality, the data was time aligned and scaled as follows:

1. The BMD has a built in time delay of 2.5 seconds on the 3rd control signal w.r.t. the DVE, to properly account for the sample transport lag time in the BMD plumbing. The 3rd MFC data was shifted by 3 seconds to account for this. Because this data was only available to 1-second time resolution, we could not shift it exactly 2.5 seconds. The DVE unit has almost instantaneous response to exhaust volume changes and therefor was not time shifted.

2. The DVE data was scaled to be as close as possible to the magnitude of the BMD's 3rd MFC data. This scaling is done by phase, with phases 1 and 3 having a multiplier of 1.6, and phase 2 having a multiplier of 1.3. The reason two multipliers are used is to account for the reduced flow rate into the sample bags of the 3rd MFC in phase 2. Similarly, the scaling of the US06 tests was 0.92. Again, this scaling was done to make comparisons as easy as possible.

In addition a regression analysis was done on all 1 second data, comparing the DVE flow signal to the 3rd MFC flow signal. This regression of data points demonstrates good proportionality between the DVE and the 3rd MFC, with a linear correlation coefficient (R^2) of typically 0.99.

Note that in phase III of FTP#1, a series of hard acceleration/decelerations were performed where the driver did not follow the FTP trace. This was done to further look at time response of the system under sever driving conditions. Again the data demonstrates good proportionality between the DVE and the 3rd MFC.

Summary

Based on the control system model used, the DVE and 3rd MFC should track actual vehicle exhaust volume fairly well, and actual vehicle test data supports this premise. One problem in this analysis was that 1/10th second data. was not available. If 1/10th second data were available, it is believed the agreement would have been even better.

Proportionality Data – FTP #1

Figure 1 - Time Aligned Plot
DVE (scaled) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase#1&2

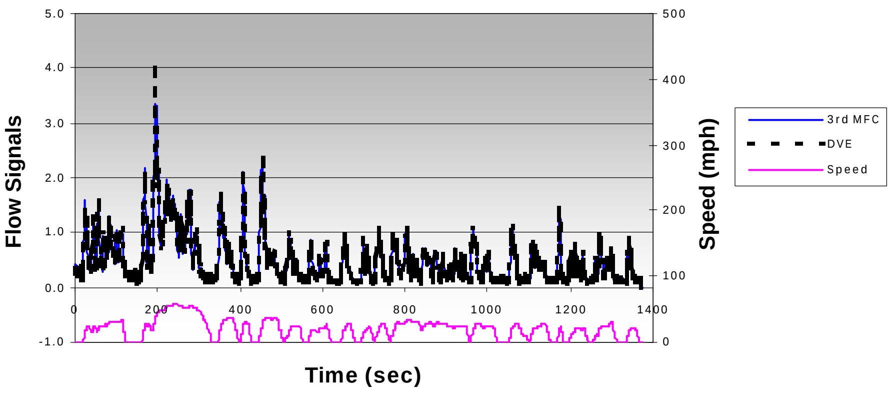


Figure 2 - Time Aligned Plot
DVE (scaled by 1.6) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase #1 (0 - 250 sec)

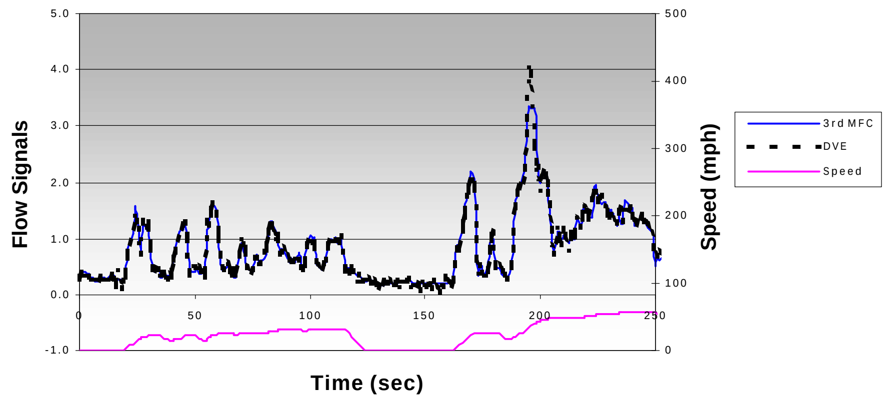


Figure 3 - Time Aligned Plot
DVE (scaled by 1.6) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase #1 (250 - 500 sec)

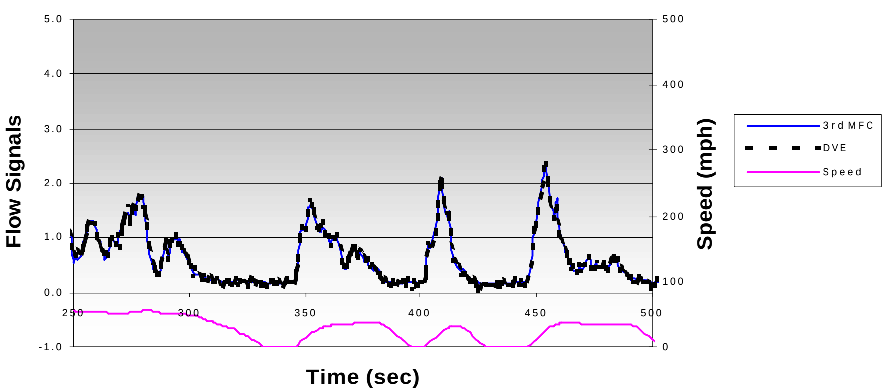
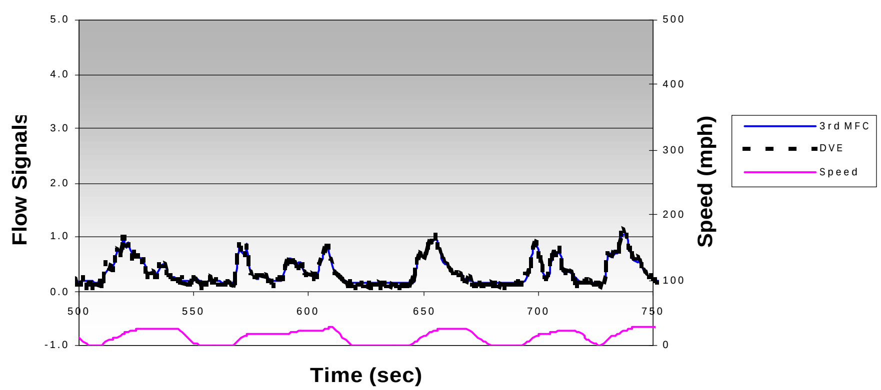


Figure 4 - Time Aligned Plot
DVE (scaled by 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase #2 (500 - 750 sec)



Proportionality Data – FTP #1

Figure 5 - Time Aligned Plot
DVE (scaled by 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase #2 (750 - 1000 sec)

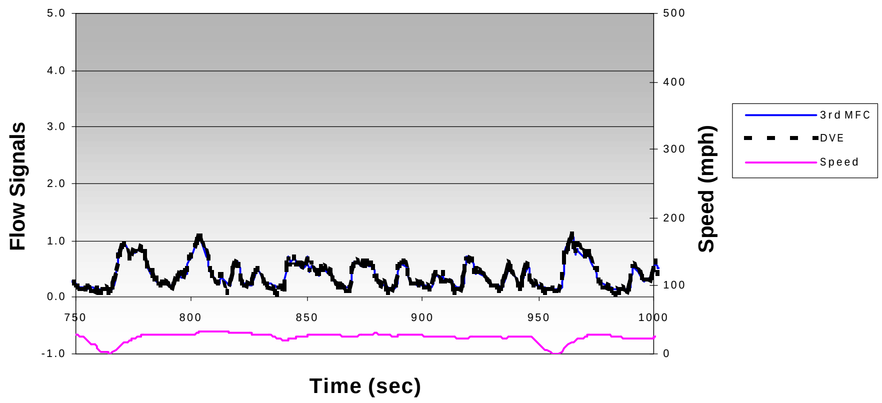


Figure 6 - Time Aligned Plot
DVE (scaled by 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase #2 (1000 - 1250 sec)

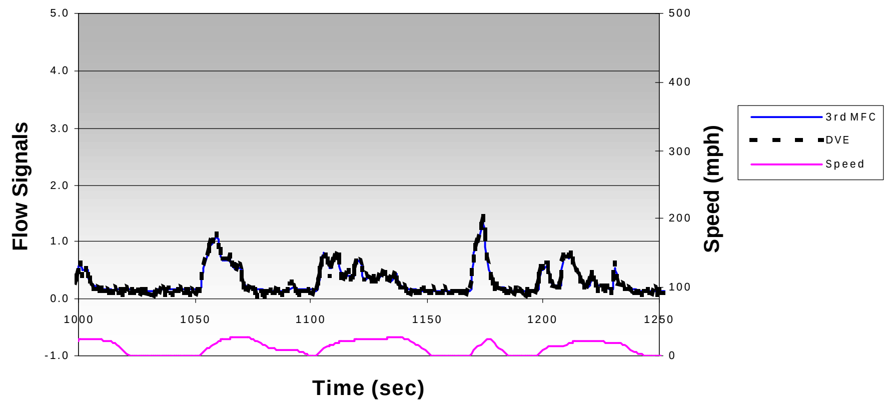


Figure 7 - Time Aligned Plot
DVE (scaled by 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase #2 (1250 - 1372 sec)

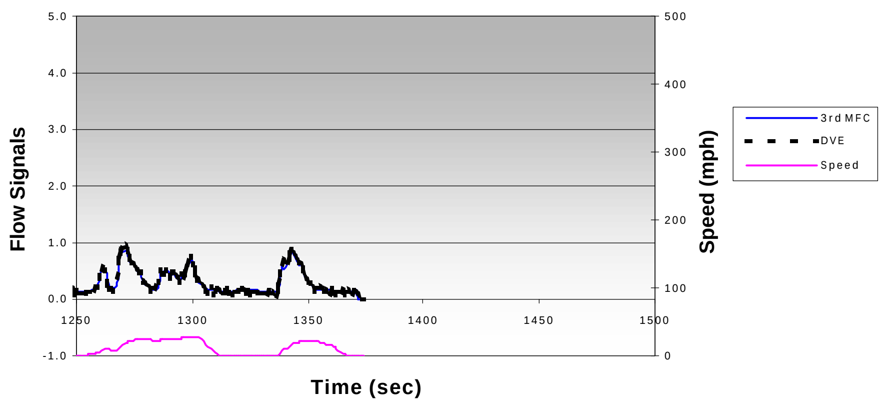
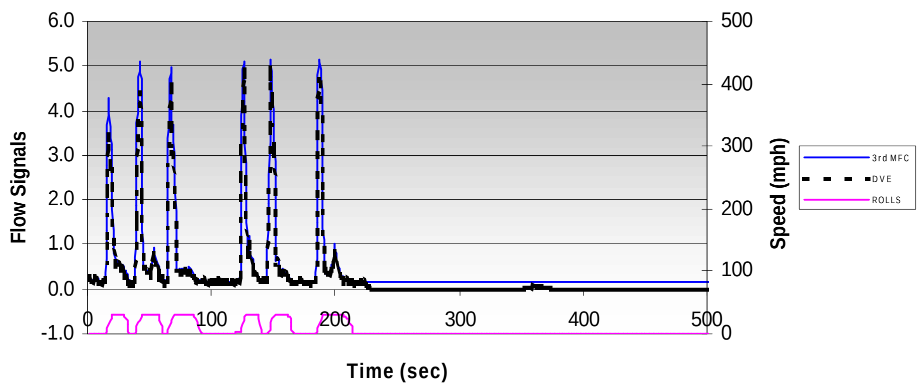
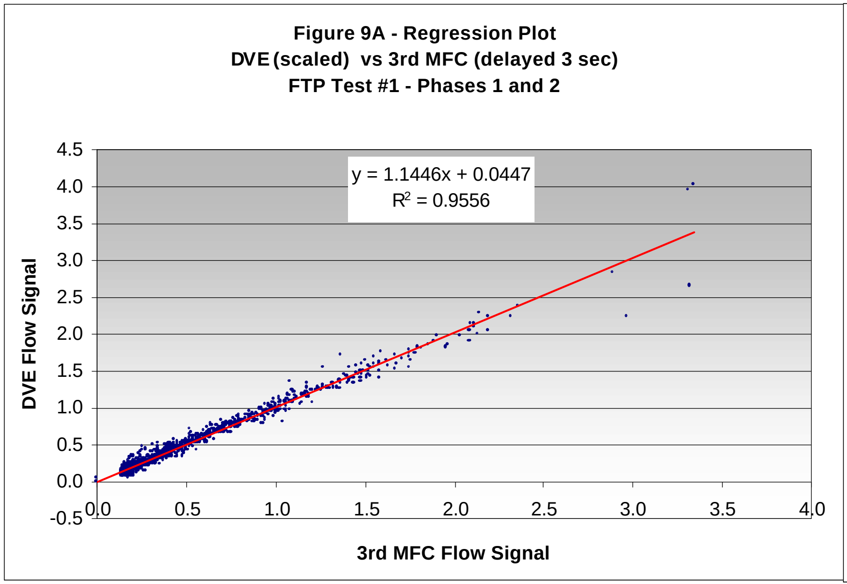
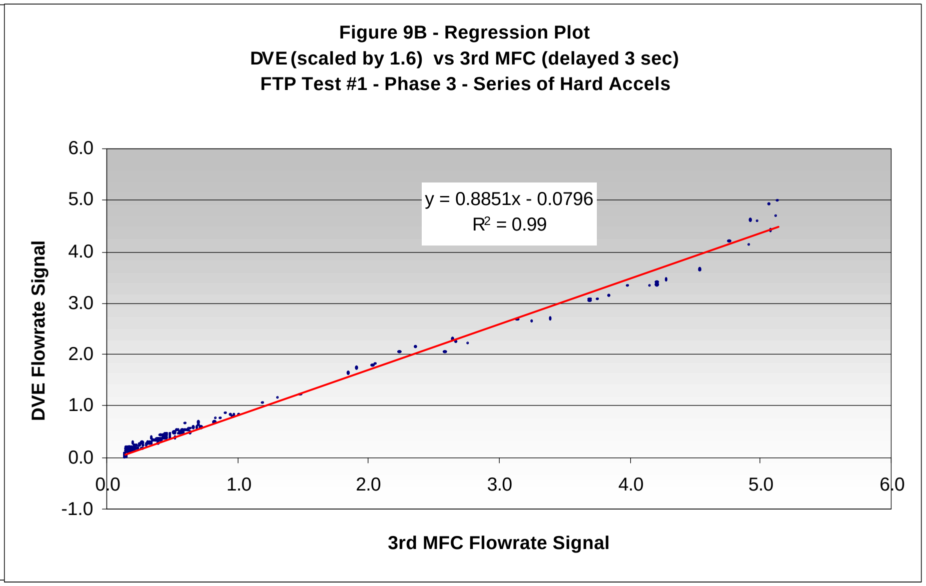


Figure 8 - Time Aligned Plot
DVE (scaled by 1.6) vs 3rd MFC (delayed 3 sec)
FTP Test #1 - Phase 3 - Series of Hard Accels



Proportionality Data – FTP #1





Proportionality Data – FTP #2

Figure 10 - Time Aligned Plot
DVE (scaled) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phases 1 and 2

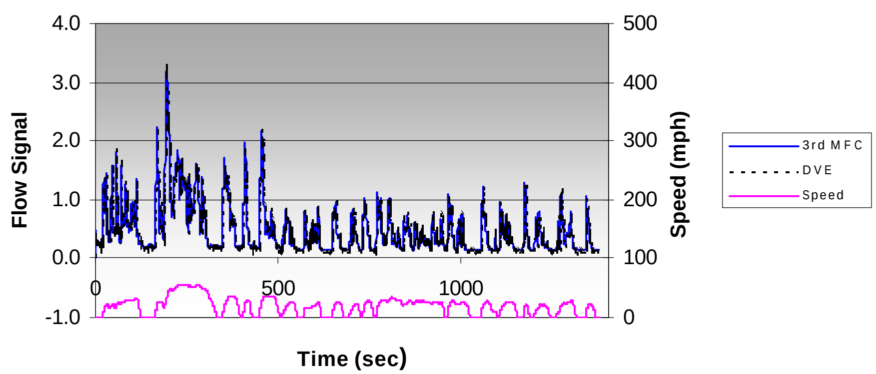


Figure 11 - Time Aligned Plot
DVE (scaled by 1.6) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phase 1 (0 - 250 sec)

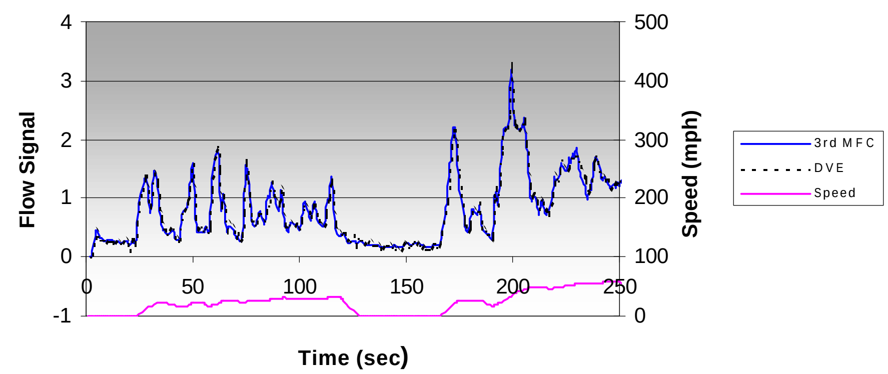


Figure 12 - Time Aligned Plot
DVE (scaled by 1.6) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phase 1 (250 - 500 sec)

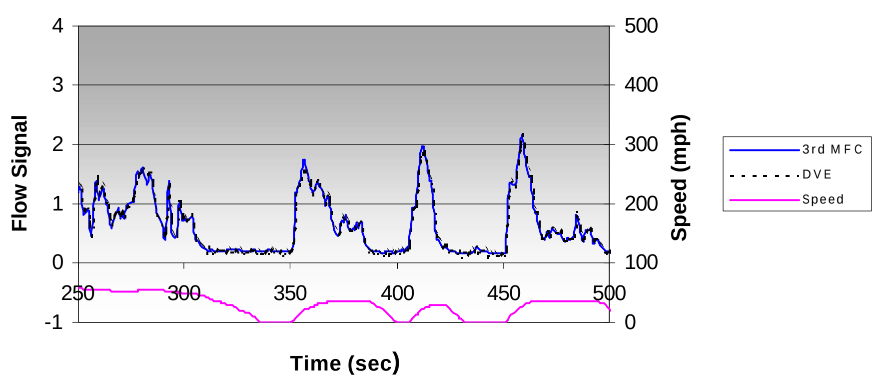
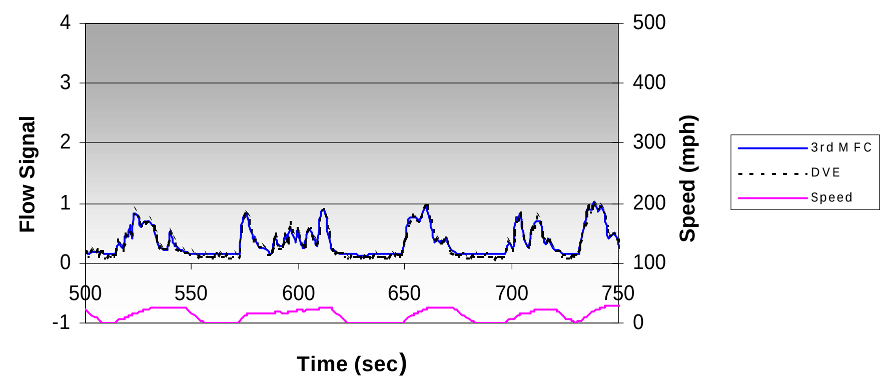


Figure 13 - Time Aligned Plot
DVE (scaled 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phase 2 (500 - 750 sec)



Proportionality Data – FTP #2

Figure 14 - Time Aligned Plot
DVE (scaled 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phase 2 (750 - 1000 sec)

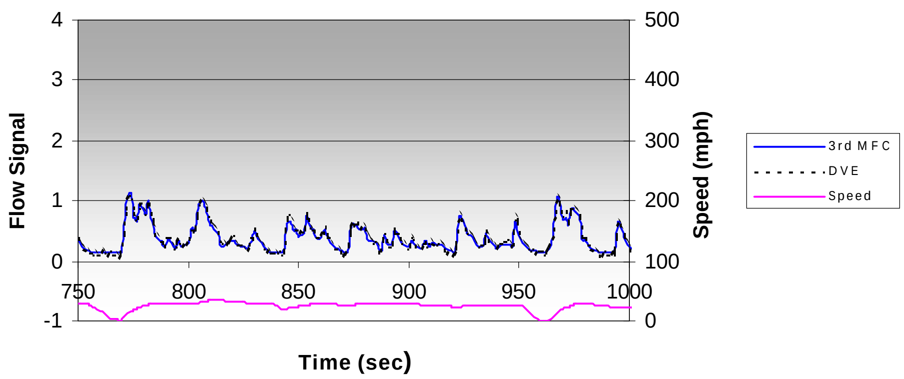


Figure 15 - Time Aligned Plot
DVE (scaled 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phase 2 (1000 - 1250 sec)

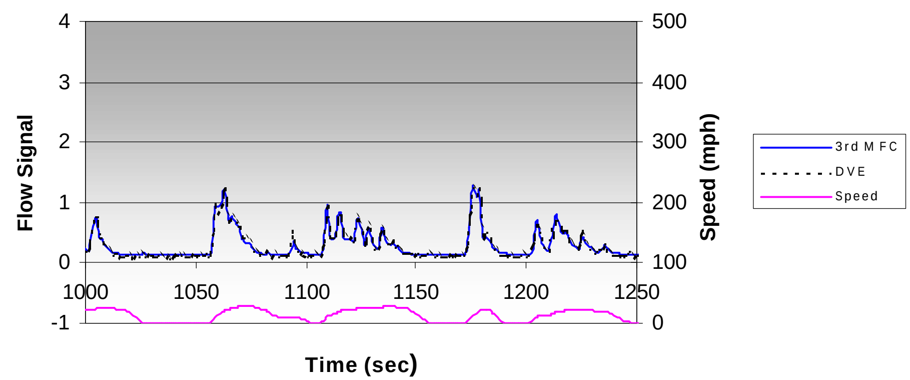


Figure 16 - Time Aligned Plot
DVE (scaled 1.3) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phase 2 (1250 - 1372 sec)

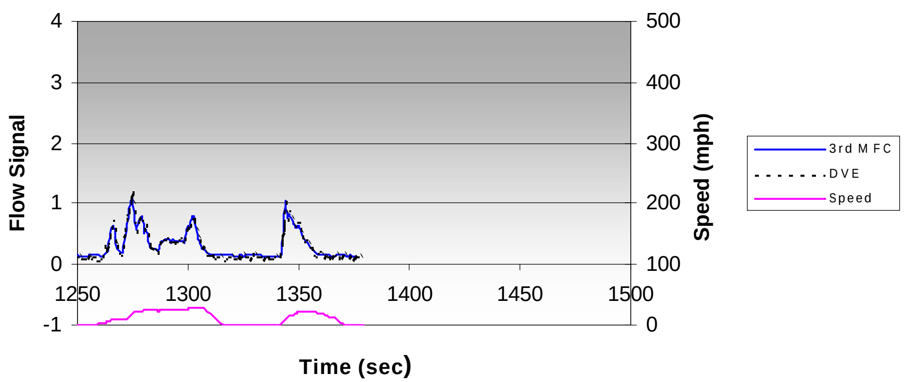
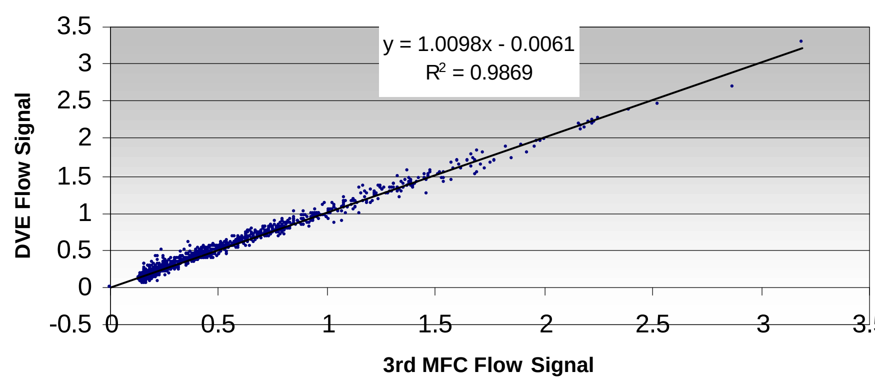


Figure 17 - Regression Plot
DVE (scaled) vs 3rd MFC (delayed 3 sec)
FTP Test #2 - Phase 1&2



Proportionality Data – US06 #1

Figure 19 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#1

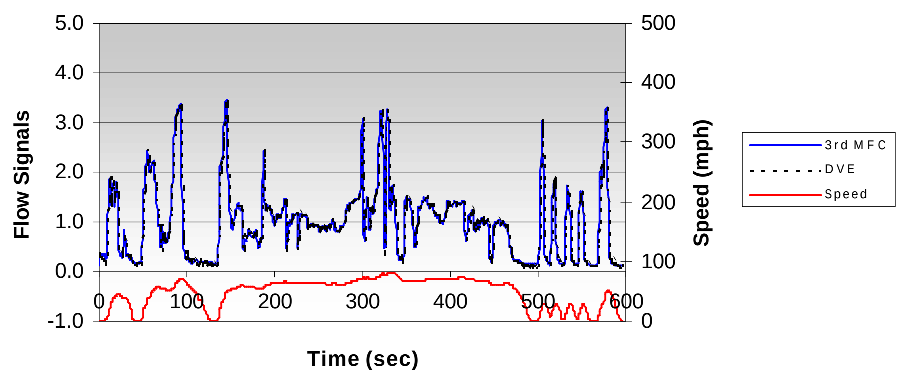


Figure 20 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#1 (0 - 250 sec)

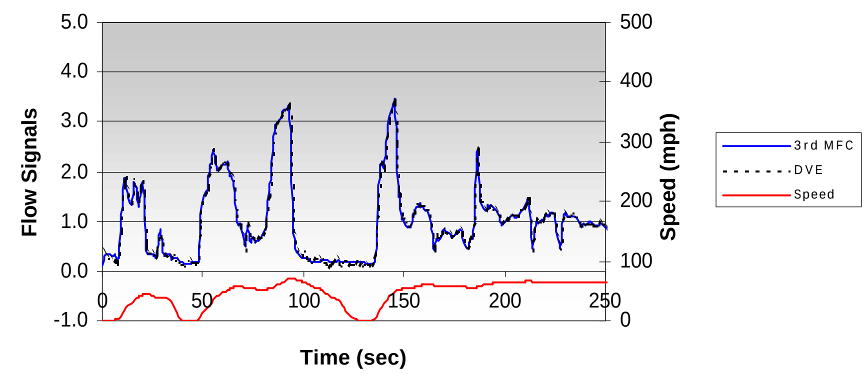


Figure 21 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#1 (250 - 500 sec)

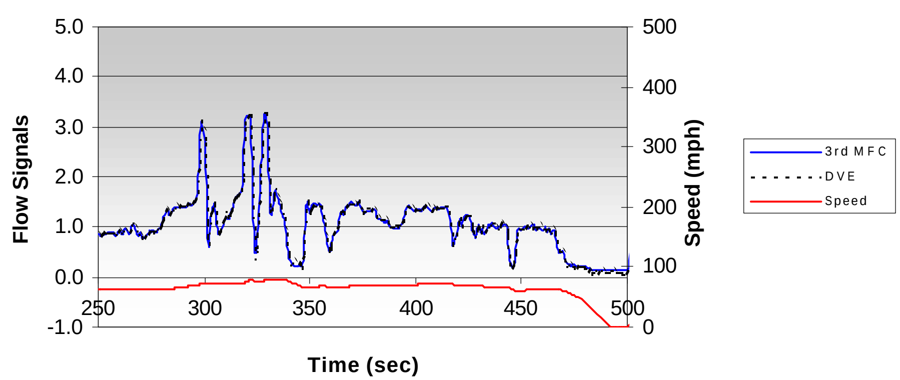
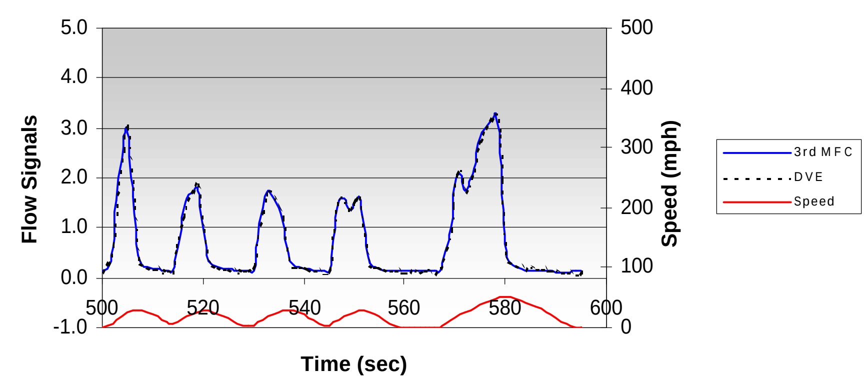
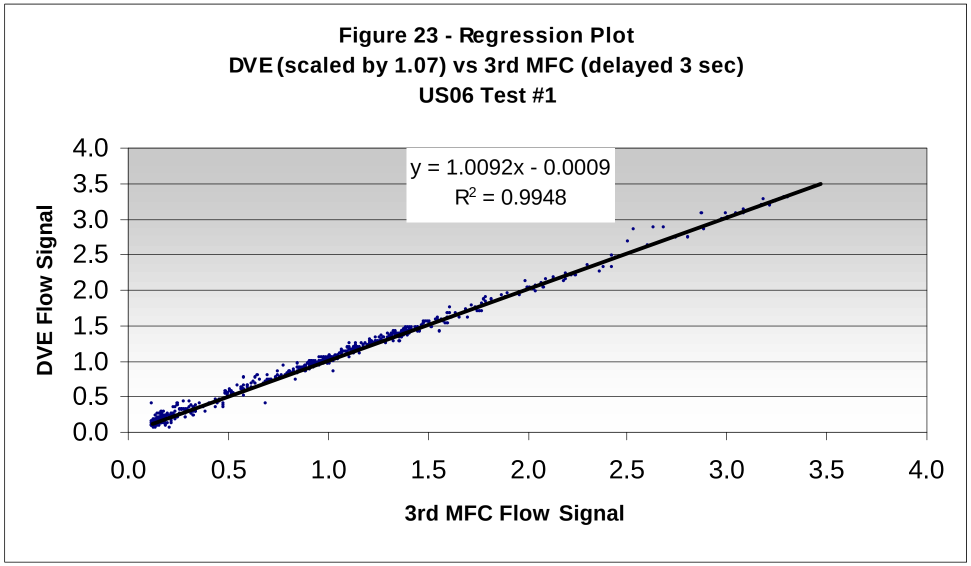


Figure 22 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#1 (500 - 600 sec)



Proportionality Data – US06 #1



Proportionality Data – US06 #2

Figure 24 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#2

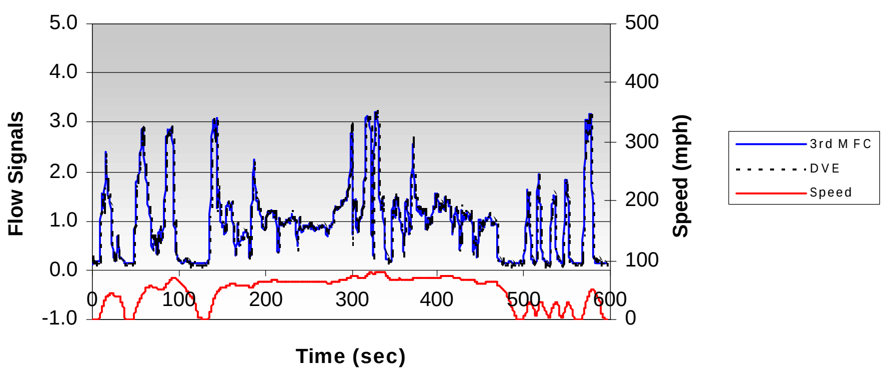


Figure 25 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#2 (0 - 250 sec)

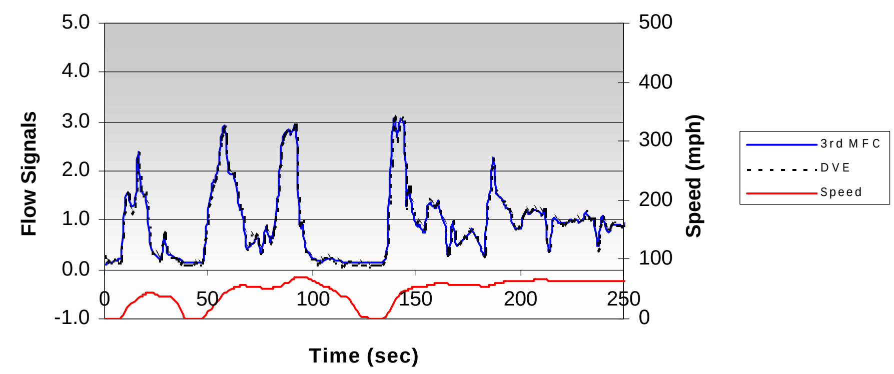


Figure 26 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#2 (250 - 500 sec0

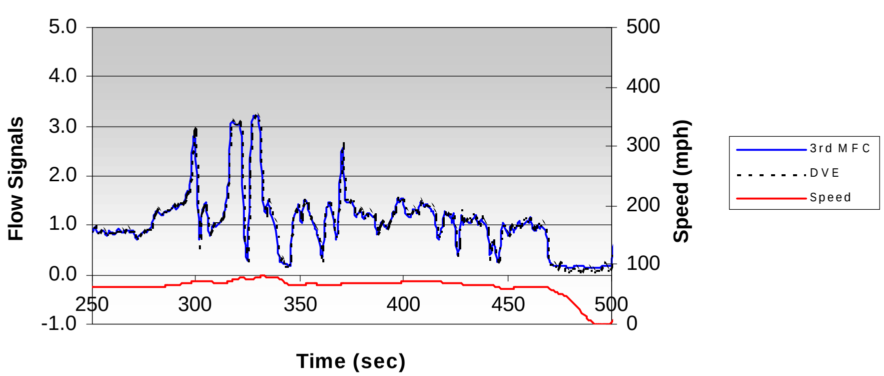
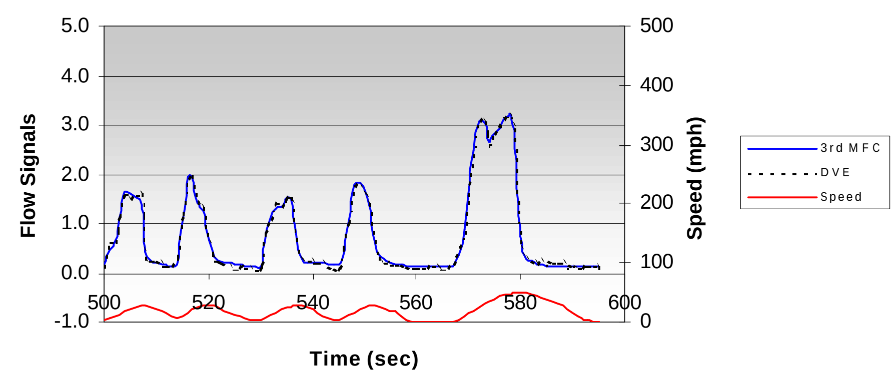


Figure 27 - Time Aligned Plot
DVE (scaled by 1.07) vs 3rd MFC (delayed 3 sec)
US06 Test#2 (500 - 600 sec)



Proportionality Data – US06 #2